More

Two-Slit Interference Pattern

The meaning of wave-particle duality can be illustrated clearly by again considering the double-slit interference pattern along the lines of a discussion first given by R. P. Feynman.[13] We shall consider the case of electrons falling on the two slits, though the analysis would be identical for light. The experimental arrangement is shown in Figure 5-23*a*. (This is also a gedankenexperiment—don't try this at home!) The accelerating voltage provides all electrons with essentially the same energy, hence the same wavelength λ. The electron detector can be moved vertically along the wall so that the number of electrons arriving at the detector can be recorded as a function of the angle θ, allowing a measurement of the number of electrons per minute (the counting rate of the detector) arriving at each point along the wall. As the experiment proceeds, two things become apparent. (1) The detector either records the arrival of an electron at the wall, or it does not. Specifically, it sees no "half electrons" or "partial electrons." (2) The detector counting rate varies along the wall; i.e., the probability that the detector will observe an electron varies with the angle θ. The result of the experiment is the probability curve P_{12} in Figure 5-23*b*, which is the number of electrons/minute (intensity) versus the location along the wall.

Now we would like to analyze the curve in Figure 5-23*b* to see if we understand the behavior of electrons. Since the detector sees only discrete particles, i.e., whole electrons, not partial ones, then in order to reach the detector one might think an electron has passed through either slit 1 or slit 2. On this basis, all of the electrons reaching the wall go through one or the other of the two slits, so our observed curve P_{12} must be the sum of the effects of the electrons passing through slit 1 and those passing through slit 2. We can check this prediction by blocking slit 2 and measuring the counting rate along the wall with only slit 1 open. The result is curve P_1 in Figure 5-23*c*. Then we repeat the measurement with slit 1 blocked and only slit 2 open. The result of that experiment is curve P_2 in Figure 5-23*c*. Clearly, P_{12} made

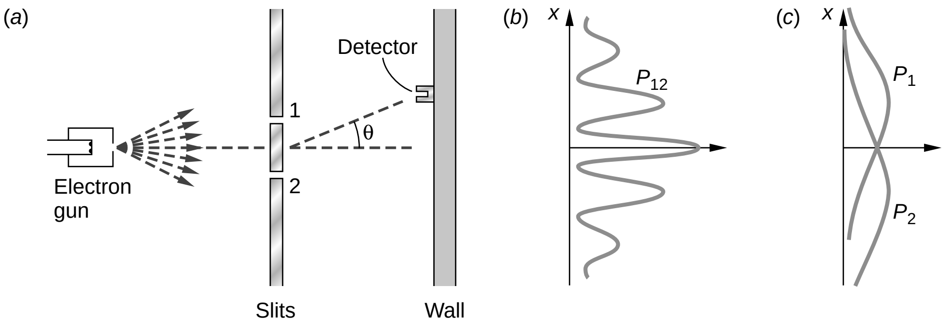

Fig. 5-23 (*a*) Experimental arrangement for producing a double-slit diffraction pattern with electron waves. The detector can move up and down the wall. (*b*) The probability distribution P_{12} measured with both slits open. (*c*) The probability distributions P_1 and P_2 measured with only P_1 and only P_2 open, respectively.

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with both slits open is *not* the sum of P_1 and P_2, the counting rates or probabilities for electrons passing through each slit alone; i.e., $P_{12} \neq P_1 + P_2$.

In analogy with our experience with other kinds of waves, e.g., light and water, we recognize this to be the result of interference. P_{12} is the double-slit interference pattern formed by the electron waves. The pattern has its maximum at $\theta = 0°$, and the first minimum at θ given by $d \sin \theta = \lambda/2$, where d is the separation of the slits. If we take θ to be small, as is usually the case in such experiments, then we can write $\theta \approx \lambda/2d$ for the position of the first minimum. How does the interference come about? Just as it does for classical waves, where interference between two (or more) waves arises by adding the amplitudes of the waves, taking their relative phases into account, and then squaring the result to obtain the intensity. Thus, if the wave functions of the electron waves are Ψ_1 at slit 1 and Ψ_2 at slit 2, then we have for the three curves in Figure 5-23*b* and *c*

$$P_1 = |\Psi_1|^2 \qquad P_2 = |\Psi_2|^2 \qquad \text{and} \qquad P_{12} = |\Psi_1 + \Psi_2|^2 \tag{5-32}$$

Thus, we see that electrons are detected as particles but have propagated through space as a wave. In other words, the electrons behave as particles only when we look at them! This is what we mean by wave-particle duality. This result is known as Bohr's principle of complementarity—the particle aspects and wave aspects complement each other. Both are needed, but both cannot be observed at the same time. Whether the wave aspect or the particle aspect is observed depends upon the experimental arrangement. There are many subtleties associated with the fact that nature works this way. Let's examine one of them before we leave this topic.

Using the same experimental setup as before, we will modify our gedankenexperiment slightly so as to enable us to have both slits open, but watch the electrons to see which slit each goes through. We'll do this by installing a light source behind the slits as shown in Figure 5-24*a*. Since charged particles scatter light, an electron going through slit 2 along the dashed line to the detector would scatter some light near *A* and we would see the flash in the vicinity of slit 2. Similarly, an electron passing through slit 1 will result in a flash in the vicinity of slit 1. When we do the experiment, here is what happens: whenever the detector records an electron, we also see a light flash either in the vicinity of slit 1 or in the vicinity of slit 2, but *never* near both at once; i.e., the electrons don't go partly through slit 1 and partly through slit 2. The same is always true, no matter where along the wall the detector is located. This result

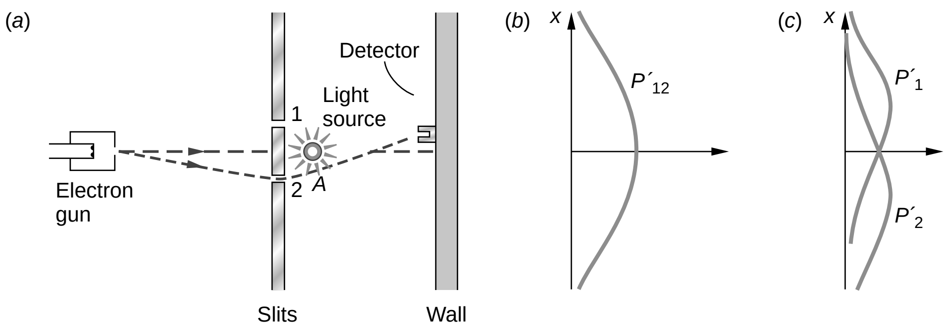

Fig. 5-24 (*a*) A light source is added to provide a means of seeing which slit the electrons pass through. (*b*) The probability distribution P'_{12} measured with both slits open, but a determination as to which slit each electron went through. (*c*) The distribution of electrons observed to pass through each slit.

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says that when we *"look"* at the electron (i.e., scatter light from it as at *A*), we can tell which slit a particular electron went through. This is at odds with Equation 5-32, so let's look at the data carefully.

We record data to keep track of the counting rate of electrons arriving at each location of the detector and whether each went through slit 1 or slit 2, according to where the light flash occurred. From the number that went through slit 1, the data result in the curve P_1' in Figure 5-24*c;* and from those that went through slit 2, we obtain curve P_2'. These look just like curves P_1 and P_2 in Figure 5-23*c,* which were made with the other slit closed, and that is what we expect. However, when graphing the counting-rate data for all of the electrons that went through both slits, curve P_{12}' in Figure 5-24*b* results where now P_{12}' *is* the sum of P_1' and P_2' and the double-slit diffraction pattern has disappeared! By "looking" at the electrons as they came through the slits, we have changed their motion; e.g., an electron that may have gone to a P_{12} maximum, after being bumped by the light, may end up at a P_{12} minimum instead. This, the observation of the electrons, is what destroyed the double-slit pattern. Speaking a bit more quantitatively, determining that the electron went through a particular one of the slits means that we have localized its position to within $\Delta x \approx d/2$, where d is the slit separation. The uncertainty principle then gives

$$\Delta p\, \Delta x \approx \Delta p\, d/2 \geq \hbar$$
$$\text{or} \qquad \Delta p \geq 2\hbar/d$$

Thus, if an electron was originally heading toward the interference maximum of P_{12} at $\theta = 0°$ with momentum $p = \lambda / h$, it will be deflected by the light-scattering event through an angle which is uncertain by an amount $\Delta\theta$ where

$$\Delta\theta \approx \frac{\Delta p}{p} = \frac{(2\,\hbar/d)}{(h/\lambda)} = \frac{\lambda}{\pi d}$$

which, as we noted earlier, is about the position of the first minimum of the diffraction pattern. Thus, the *observation* of the electron washes out the pattern.