More

Thermal Conduction

Good conductors of electricity are also good conductors of heat. The classical theory assumes that this is because the electron gas is mainly responsible for heat conduction in metals. The coefficient of heat conduction or thermal conductivity K of a solid is defined in terms of the steady-state flow of thermal energy along a rod along which there is a uniform temperature gradient dT/dx. The flux of thermal energy J_Q, the net energy flow across a unit area per unit time, is then

$$J_Q = -K\frac{dT}{dx} \tag{10-20}$$

Kinetic theory, reviewed in Section 8-1, enables us to determine K in terms of the classical characteristics of the electron gas. Consider a portion of a metal rod as shown in Figure 10-12. The number of electrons moving in the $+x$ direction is $(nv_x)/2$, where n is the density of electrons and v_x is the average value of the $+x$ components of their velocities. On the average, an equal number are moving in the $-x$ direction. If the heat capacity of a single electron is c, then in moving from left to right through the rod, that is, from the region where the temperature is $T + \Delta T$ to a region where the temperature is T, the electron will lose $c\,\Delta T$ of thermal energy. Over a distance equal to the mean free path λ,

$$\Delta T = \frac{dT}{dx}\lambda_x = \frac{dT}{dx}v_x\tau \tag{10-21}$$

where λ_x is the x component of λ in the figure and τ is the collision time. The net flux of thermal energy is the product of the total flux of electrons (in both x directions) nv_x and the thermal energy change $c\,\Delta T$ for each electron, or

$$J_Q = -nv_x c\,\Delta T = -nv_x^2 c\tau\frac{dT}{dx} \tag{10-22}$$

Recalling that the equipartition theorem (Section 8-1) implies that $v_x^2 = (1/3)\langle v^2\rangle$, the above can be written as

$$J_Q = -\frac{1}{3}n\langle v^2\rangle c\tau\frac{dT}{dx} \tag{10-23}$$

The heat capacity per unit volume C is equal to nc and $\tau = \lambda/\langle v\rangle$, so Equation 10-23 becomes

$$J_Q = -\frac{1}{3}C\langle v\rangle\lambda\frac{dT}{dx} \tag{10-24}$$

which, when compared with Equation 10-20, yields for the thermal conductivity

$$K = \tfrac{1}{3}C\langle v\rangle\lambda \tag{10-25}$$

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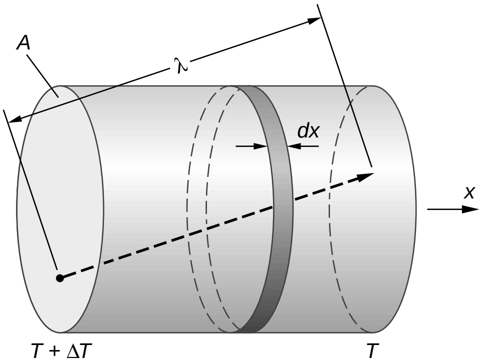

Fig. 10-12 An electron moving a distance λ between collisions gives up energy equal to $c\,\Delta T$, where $\Delta T = (dT/dx)\lambda_x$.

The heat capacity per unit volume C can be expressed in terms of the molar heat capacity at constant volume of the electron gas C_v by noting that $nC_v/N_A = C \cdot C_v$ for a monatomic gas is given by $C_v = (3/2)R = (3/2)kN_A$ (see Section 8- 1). Using these, the thermal conductivity can be written as

$$K = \tfrac{1}{2}n\langle v\rangle\lambda k \qquad \textbf{10-26}$$

Thus, K and the electrical conductivity σ are related by

$$\frac{K}{\sigma} = \frac{\frac{1}{2}n\langle v\rangle\lambda k}{ne^2\lambda/m_e\langle v\rangle} = \frac{m_e\langle v\rangle^2 k}{2e^2} = \frac{4k^2T}{\pi e^2} \qquad \textbf{10-27}$$

where we have used Equation 10-9 for $\langle v\rangle$.

The classical theory therefore predicts that the ratio of thermal to electrical conductivity is proportional to the absolute temperature and that the proportionality constant is the same for any metal. Surprisingly, it does not depend on either n or m_e. This is known as the *Wiedermann-Franz law.* The ratio $K/\sigma T$ is called the *Lorentz number:*

$$L = \frac{K}{\sigma T} = \frac{4k^2}{\pi e^2} \approx 1.0 \times 10^{-8}\ \mathrm{W \cdot \Omega / K^2} \qquad \textbf{10-28}$$

Table 10-2 shows that $K/\sigma T$ is indeed nearly the same for all metals and is independent of temperature, though the numerical value is somewhat higher than predicted.[5] Because of the simplicity of the model, we can only hope for an order-of-magnitude

Table 10-2 Lorentz number $L = K/\sigma T$ in units of $10^{-8}\ \mathrm{W \cdot \Omega/K^2}$, for several metals at 0°C and 100°C

Metal	0°C	100°C	Metal	0°C	100°C
Ag	2.31	2.37	Pb	2.47	2.56
Au	2.35	2.40	Pt	2.51	2.60
Cd	2.42	2.43	Sn	2.52	2.49
Cu	2.23	2.33	W	3.04	3.20
Mo	2.61	2.79	Zn	2.31	2.33

Data from Kittel (1995).

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agreement. The important test of the model is that, though K and σ vary greatly with temperature and from metal to metal, the ratio $K/\sigma T$ does not. This result was very important in the development of the theory of metals, since it gave strong support to the idea of an electron gas permeating the lattice of metal ions.

Questions

5. In the classical free-electron model the electron loses energy (on the average) in a collision, since it loses the drift velocity it has picked up since the last collision. Where does this energy appear?

6. At low temperatures the Lorentz number tends to decrease. Why might such a decrease occur?
