CHAPTER
22

IN THIS CHAPTER WE COVER . . .

22.1 Simple Interest
22.2 Compound Interest
22.3 Conventional Loans
22.4 Annuities

Borrowing Models

In the previous chapter, we looked at consumer financial models for saving and formulas for calculating the amount accumulated. Savings or investments would not earn interest unless they could be loaned to someone to make productive use of the money.

In this chapter, we examine the other side of consumer finance, borrowing. You are likely to borrow to buy a car, you will almost certainly borrow if you buy a house or apartment, and you are borrowing if you use a credit card. You will pay interest (and perhaps finance charges) in addition.

But there are different ways to structure loans that can make a lot of difference in how much you pay for the use of the money that you borrow. We investigate and compare some common kinds of loans.

We briefly (re)acquaint you with compound interest and a few formulas from Chapter 21. If you have a grasp of the ideas behind compound interest and can use the formulas, you can proceed with this chapter without first reading Chapter 21.

22.1 Simple Interest

The amount of **interest** charged on a loan is determined by the **principal,** by the amount borrowed, and by the method used to calculate the interest. With **simple interest,** the borrower pays a fixed amount of interest for each period of the loan. The interest rate is usually quoted as an annual rate.

In the previous chapter (p. 806), we cited the example of simple interest paid on a bond. We do not repeat the example here, but proceed to consumer loans. First, we remind you of the relevant formula, now in the context of loans rather than savings.

For a principal P and an annual rate of interest r, the interest owed after t years is

$$I = Prt$$

and the total amount A due on the loan is

$$A = P(1 + rt)$$

Add-On Loan

A common type of consumer loan is the *add-on* loan. You borrow an amount P to be repaid in t years. The interest is simple interest at an annual rate r (= $100r$%), for a total of $I = Prt$. You must pay $P + I = P(1 + rt)$ in installments (usually monthly). With n payments, each payment is $d = P(1 + rt)/n$.

Add-on Loan

By the terms of an **add-on loan** over n installments, you pay $\frac{1}{n}$th of the principal and $\frac{1}{n}$th of the total interest with each payment. The total interest is "added on" to the principal, and the sum is paid in equal installments.

Add-On Loans — EXAMPLE 1

Suppose that you have to borrow \$8000 to buy a used car. The dealer offers you a 5% add-on loan to be repaid in monthly installments over four years. This sounds like a much better deal than the 8% loan that you can get at the credit union. How much is your payment d on the dealer's add-on loan?

SOLUTION To find the total amount to be paid over the four years, first calculate the interest on the add-on loan:

$$I = Prt$$
$$I = \$8000(0.05)(4)$$
$$I = \$1600$$

The total amount that must be repaid is then $P + I = \$8000 + \$1600 = \$9600$, and the monthly payment over 4 years (48 months) is $\$9600 \div 48 = \$200$. ■

With an add-on loan, everything sounds simple and straightforward (even the calculation of the payment!). The interest is calculated on the entire principal. Because you slowly pay back the principal, however, you do not have the use of the whole amount for the entire duration of the loan. In fact, you have the use of the full principal for just one month. You do, however, have the use of the car! But the net value that you have at any point is the cost of the car minus the amount of principal already repaid (here we neglect depreciation of the car). It is on this net value, and the amount of interest, that the interest rate should be calculated. In effect, the "true" interest rate (we will make this concept more precise) is much higher than the rate r quoted.

Discounted Loan

Another type of consumer loan is a *discounted loan*. The interest is computed as simple interest, just as for an add-on loan, but it is subtracted from the amount given to the borrower. In other words, instead of getting the principal P, the borrower gets the *proceeds* $P - I = P - Prt = P(1 - rt)$ but must pay back the amount P over the term of the loan. In effect, the interest is paid in advance. What is discounted is not the cost of the loan but how much the borrower gets! The interest is based on the entire P, but the borrower never has the use of that much and, as with an add-on loan, is paying interest on the entire amount P but has the use of less and less of it as the loan is repaid.

Discounted Loan

With a **discounted loan** over n installments, you pay $\frac{1}{n}$th of the principal with each payment. The amount that you receive from the lender is the principal "discounted by" (minus) the total interest, but you pay back the entire principal in equal installments.

EXAMPLE 2 Discounted Loans

Suppose again that you have to borrow \$8000 to buy a used car. Your neighborhood loan office offers you a 5% discounted loan to be repaid in monthly installments over four years. Is this a better deal than the dealer's offer of a 5% add-on loan over four years (see Example 1)?

SOLUTION The total amount to be paid over the four years is $P = \$8000$ over 48 months, for a monthly payment of \$166.67. This is indeed a lower payment, but the problem is that the lender loans you only $\$8000 \times (1 - 0.05 \times 4) = \$8000 \times 0.80 = \$6400$, which isn't enough to buy the car. How big a discounted loan do you need? Call the amount x. You need $x(0.80) = \$8000$, or $x = \$8000/0.80 = \$10{,}000$. Your monthly payment is $\$10{,}000/48 = \$208.33$, so this loan is indeed cheaper than the dealer's loan. ■

22.2 Compound Interest

Compounding is the calculation of interest on interest. A common example is the balance on a credit card. As long as there is an outstanding balance owed, the interest owed is calculated on the entire balance, including any part of it that was interest calculated and added to the balance in earlier months.

Credit-Card Interest

EXAMPLE 3

Suppose that you owe \$1000 on your credit card, the company charges 1.5% interest per month, and you just let the balance ride. How much interest do you pay in the first year?

SOLUTION Your interest the first month is 1.5% of \$1000, or $0.015 \times \$1000 = \$15$. The new balance owed is $(1 + 0.015) \times \$1000 = \$1015$. Your interest the second month is not 1.5% of \$1000, or \$15 (as would be the case for simple interest), but 1.5% of \$1015, or $0.015 \times \$1015 = \$15.23$, so the new balance is

$$(1 + 0.015) \times \$1015 = \$(1 + 0.015) \times (1 + 0.015) \times \$1000 = \$1030.23$$

(We neglect the extra charges for your failure to make minimum payments.) After 12 months of letting the balance ride, it has become

$$(1.015)^{12} \times \$1000 = \$1195.62$$

In other words, the actual interest for the year comes to \$195.62, which is 19.562% of \$1000. So, although the quoted rate of interest is 1.5% per month, which seems as if it should amount to $12 \times 1.5 = 18\%$ per year, the interest owed is actually more. ■

We apply two formulas from Chapter 21: the **compound interest formula** (p. 810) and the **savings formula** (p. 821), here phrasing them for loans.

Compound Interest Formula

If a principal P is loaned at interest rate i per compounding period, then after n compounding periods (with no repayment) the amount owed is

$$A = P(1 + i)^n$$

This formula just generalizes what we saw happen with the credit-card balance.

Borrowing from Joe, the Patient Loan Shark

EXAMPLE 4

Suppose that you arrive on campus without enough money to buy your textbooks (including this one), not to mention other supplies and necessities (such as late-night pizzas). So you go to Joe's Friendly Loan Service at a nearby off-campus location, where Joe offers to loan you \$1000 at an incredible interest rate of 1% per week, compounded weekly. Joe is such a good sport that he doesn't want the money back until a year from now, which gives you a chance to earn the repayment over the summer. How much will you have to pay back at the end of a year?

SOLUTION We have $P = \$1000$, $i = 1\% = 0.01$ per compounding period, and $n = 52$ compounding periods:

$$A = P(1 + i)^n = \$1000 \times (1 + 0.01)^{52} = \$1677.69$$

EXAMPLE 5 Saving Up to Pay Joe Back

Summer comes and it's time to save up to pay Joe back. Suppose that you work for three months and at the end of each month deposit an amount d to a savings account that pays 3% interest per year, or 0.25% per month. How much does d need to be to pay off the \$1667.69 that you owe?

SOLUTION We have $A = \$1667.69$, $i = 0.25\% = 0.0025$ per compounding period, and $n = 3$ compounding periods:

$$d = \frac{Ai}{(1 + i)^n - 1} = \frac{\$1667.69 \times 0.0025}{(1 + 0.0025)^3 - 1} = \$554.51$$

Terminology for Loan Rates

We have seen that the rate of interest for a loan depends on whether or not compounding is done and how the interest is calculated. Just like the Truth in Savings Act mentioned in Chapter 21 (p. 808), the Truth in Lending Act establishes terminology and calculation methods for interest. We remind you of it here briefly.

A **nominal rate** is any stated rate of interest for a specified length of time. For instance, a nominal rate could be a 1.5% monthly rate on a credit-card balance. By itself, such a rate does not indicate or take into account whether or how often interest is compounded.

Similarly, from Chapter 21 (p. 816) you are familiar with the **effective rate,** the actual percentage rate of increase for a length of time, *taking into account compounding.* It equals the rate of simple interest that would realize exactly as much interest over that length of time.

We saw that \$1000 at a yearly interest rate of 18% (a nominal rate), or 1.5% per month, compounded monthly for one year yields \$195.62 in interest owed, which is 19.562% of the original principal. Hence, the effective annual rate is 19.562%. In other words, a \$1000 loan at simple interest of 19.562% for one year would owe exactly the same interest.

Finally, when stated per year ("annualized"), the effective rate is called the **effective annual rate (EAR).**

To keep the rates straight, we use i for a nominal rate for the specified **compounding period**–such as a day, month, or year–*within which no compounding is done;* this rate is the effective rate for that length of time. For a nominal rate compounded n times per year, we have $i = r/n$. For that \$1000 credit-card balance at 18% compounded monthly, we have $r = 18\%$ and $n = 12$, so $i = 1.5\%$ per month.

A new term that the Truth in Lending Act introduces is the *annual percentage rate (APR).*

Annual Percentage Rate (APR)

The **annual percentage rate (APR)** is the rate of interest per compounding period times the number of compounding periods per year.

$$\text{APR} = i \times n$$

In the example of the credit-card balance, the interest is compounded monthly, or $n = 12$ times per year, and the interest rate for the compounding period is $i = 1.5\%$, so the APR is $12 \times 1.5\% = 18\%$. The APR is the rate that the Truth in Lending Act requires the lender to disclose to the borrower. *The APR is not equal to the EAR* (as we have already seen in the credit-card example). Spotlight 22.1 explains how and why.

22.3 Conventional Loans

A common situation that you are likely to encounter is a loan—for a house, a car, or college expenses—to be paid back in equal periodic installments. Your payments are said to **amortize** (pay back) the loan. In these so-called **conventional loans,** each payment pays the current interest and also repays part of the principal. *As the principal is reduced, there is less interest owed, so less of each payment goes to the interest and more toward paying off the principal.*

Buying a House

EXAMPLE 6

Let's suppose that you buy a house with a \$100,000 loan to be paid off over 30 years in equal monthly installments. Suppose that the interest rate for the loan is 6.00%. How much is your monthly payment?

SOLUTION Imagine changing the setup slightly so that instead of making monthly payments, you are supposed to pay off the entire principal and interest at the end. Meanwhile, you make payments to a savings fund that you're building up to pay off the loan, and the savings fund earns the same rate of interest as the loan costs. The interest rate of 6.00% on the loan is compounded monthly, so the monthly rate is 0.5%. At the end of 30 years, the principal and interest on the loan will (by the compound interest formula) amount to

$$\$100{,}000 \times (1 + 0.005)^{30\times 12} = \$602{,}257.52$$

On the other hand, saving $\$d$ each month for 30 years at 6.00% interest compounded monthly, we know from the savings formula that you will accumulate

$$d\left[\frac{(1 + 0.005)^{360} - 1}{0.005}\right]$$

SPOTLIGHT 22.1 What's the Real Rate?

Financial experts agree that the real, "true" rate of interest for savings or loans is the effective annual rate (EAR).

The 1991 Federal Truth in *Savings* Act requires that savers be told the annual percentage yield (APY), which is the EAR.

The 1968 Federal Truth in *Lending* Act, however, requires that borrowers be told the *annual percentage rate (APR),* which is *not* the same as the EAR. The APR is the rate of interest per compounding period times the number of compounding periods per year. Thus, a credit-card rate of 1.5% per month translates to an APR of 18%. The APR does not take into account compounding. Hence it is not equivalent to—indeed, it understates—the true cost of borrowing, that is, the EAR. For the credit-card loan, with monthly compounding, the EAR is

$$(1 + 0.015)^{12} - 1 \approx 19.6\%$$

The APR also ignores costs that are sometimes involved in borrowing, such as a flat charge for making the loan in the first place ("loan-processing fee"), charges for late payments, and charges for failing to make a minimum payment.

For mortgage loans, the Truth in Lending Act requires that lenders include in the APR some of the upfront costs referred to as *closing costs:* any "loan origination" fee, "loan-processing" fee, and "points" (additional charges calculated as percentage points of the purchase price). The APR does not include title insurance, appraisal, credit-report fees, or transaction taxes.

Closing costs are paid at the closing of the sale, while interest is paid over the life of the loan. However, the APR treats the closing costs included in it as if they were amortized over the term of the mortgage, despite the fact that they are paid beforehand. Here, too, the APR understates the true costs.

But very few people hold a mortgage to its maturity. The median life of a 30-year mortgage is only about 5 years; that is, half of all mortgage holders pay off their mortgage before 5 years are up, usually because they sell the home and move elsewhere. Thus, for almost all home loans, the APR also includes interest that will never be paid.

Finally, we must take into account inflation. One advantage of buying a home with a fixed-rate mortgage is that your payment stays the same but your earnings and the value of your home are likely to go up with inflation: You are thus paying back the loan with dollars of lesser value. For any loan in a time of inflation, *Fisher's effect* (Chapter 21, p. 828) comes into play: If your loan has an EAR of 7% but inflation is running at 3.5% per year, the true cost to you of the loan is not exactly 7% − 3.5% = 3.5%. Instead, for an EAR of r and an inflation rate of a, the cost of the loan at the beginning of the first year is indeed $r - a$ (= 3.5% in our example), but at the end of the first year it is

$$c = \frac{r - a}{1 + a}$$

For $r = 7\%$ and $a = 3.5\%$, we get $c = 3.38\%$. The reason this is less than the expected 3.5% is that at the end of the first year you are paying back the loan with dollars that have been inflated for a year. As inflation mounts over the term of a mortgage, the cost c goes down steadily each year. For example, at the end of five years of steady inflation at 3.5%, the total inflation has been $a = (1 + 0.035)^5 - 1 = 18.8\%$, and we have $c = 2.95\%$.

A final—and major—consideration is that interest paid on your home mortgage is deductible from taxable income on federal, state, and some local income tax returns. Thus, your home ownership is subsidized by other taxpayers (just as you help subsidize home buyers among them), and the cost to you of the loan is reduced further.

To make d just the right amount to pay off the loan exactly, we need to solve the equation

$$d\left[\frac{(1+0.005)^{360}-1}{0.005}\right] = \$100{,}000 \times (1+0.005)^{30\times 12} = \$602{,}257.52$$

for the value of d, getting $d = \$599.55$ as your monthly payment. The total of the payments is "only" $360 \times \$599.55 = \$215{,}838.00$–on a loan of just \$100,000. ■

We put this idea into a more general setting.

Paying Off a Conventional Loan Is Like Saving

Let the principal be A, the effective interest rate per period i, the payment at the end of each period d, and let there be n periods. Then we have

$$A(1+i)^n = d\left[\frac{(1+i)^n - 1}{i}\right]$$

The compound interest formula is on the left and the savings formula is on the right. You can think of paying off the loan as making payments to a savings account that earns interest at the same rate as the loan. The savings balance will exactly equal the principal plus the interest on the loan at the end of the loan term.

Thus, the quantity A is sometimes called the *present value of an annuity* of n payments of d, each at the end of an interest period, with interest compounded at rate i in each period. This terminology agrees with that used in some calculators, such as the TI-83, which have a financial mode with the option to calculate A. Solving for A gives the **amortization formula.**

Amortization Formula

$$A = d\left[\frac{1-(1+i)^{-n}}{i}\right]; \qquad d = \frac{Ai}{1-(1+i)^{-n}}$$

Making Weekly Payments to Joe

EXAMPLE 7

You're worried about having to come up with the lump sum (\$1677.69) that you will owe Joe after a year, so you ask him if you can make weekly payments instead. He is delighted at the idea but says that the interest will have to be higher, 1.5% per week. He agrees to let you amortize the loan over 52 weeks, so that each payment covers the week's interest on the amount still outstanding and pays off some of the principal. How much is your weekly payment?

SOLUTION We have $A = \$1000$, $i = 1.5\% = 0.015$ per compounding period (one week), and $n = 52$ compounding periods; we want to find d:

$$d = \frac{\$1000 \times 0.015}{1 - (1 + 0.015)^{-52}} = \$27.83$$

EXAMPLE 8 Buying a Car

You decide to buy a new Wheelmobile car. After a down payment, you need to finance (borrow) \$12,000. Comparing interest rates offered by the car dealership, local banks, and your credit union, the best deal you can find is 4.9% compounded monthly over 48 months. What is your monthly payment?

We have $A = \$12{,}000$, monthly interest rate $i = 0.049/12$, and $n = 48$. Using the amortization formula, we have

$$d = \frac{\$12{,}000 \times \frac{0.049}{12}}{1 - (1 + \frac{0.049}{12})^{-48}} = \$275.81$$

How much interest do you pay? You make payments totaling $48 \times \$275.81 = \$13{,}238.88$, so the interest is $\$13{,}238.88 - \$12{,}000 = \$1{,}238.88$.

If you had bought a Plushmobile instead, with \$24,000 to finance, you would have borrowed twice as much and your monthly payment would have been twice as much.

EXAMPLE 9 APR Versus EAR for an Add-On Loan

We return to the earlier 5% add-on loan for \$8000 to be repaid over four years. The effective interest rate per month is i in the amortization formula. As hard as you try, though, you cannot solve the amortization formula for i in terms of A, d, and n. Mathematicians have shown that it cannot be done! What can you do?

SOLUTION You can use a spreadsheet and try successive values of i until you find one that makes the two sides of the equation close enough to being equal. If your spreadsheet or calculator has a Solve feature, you can use that. In this case, you find $i = 0.0077014725$.

Here we now see a distinction. The APR (see Spotlight 22.1), which the dealer is required to tell you, is 12 times the monthly rate, or $12 \times 0.0077014725 = 9.24\%$. The EAR, which takes into account monthly compounding, is $(1 + i)^{-1} = 9.64\%$.

Either way, is this a better rate than the 8% credit union loan? Before you jump to conclusions, better find out the details of that loan!

A car loan is usually for 48 or 60 months, but when you buy a home, you usually borrow a great deal more money and hence pay it off over a much longer period. The usual term for a home mortgage is 30 years. We disregard here some of the closing costs connected with buying a home and getting the loan, to focus on the loan itself.

As a simple example, consider a mortgage for $184,000 (the median cost of a U.S. home in mid-2004) at an interest rate of 5.4% over 30 years. Payments are monthly. The monthly interest rate is 5.4%/12 = 0.45% = 0.0045 and there are $30 \times 12 = 360$ months. We use the amortization formula with $A = \$184{,}000$, $i = 0.0045$, and $n = 360$:

$$d = \frac{\$184{,}000 \times \frac{0.054}{12}}{1 - (1 + \frac{0.054}{12})^{-360}} = \$1033.22$$

When you decide to buy a home, a key question that you need to ask yourself is how much home you can afford.

Thirty–Year Mortgage on Median-Priced Home

EXAMPLE 10

Let's suppose that you are a family with the U.S. median income of about $65,000 for a family of four, that you want to buy a median-priced home ($206,000 in April 2005) with a 30-year fixed-rate mortgage at 5.4%, and that you can make a down payment of only about 5%, or $10,000. Can you afford such a home?

SOLUTION Lenders have "affordability" guidelines that suggest that a family can afford to spend about 28% of its monthly income on housing. Thus, by their guidelines, you can afford $0.28 \times \$65{,}000/12 = \$1516.67$ per month.

What is the monthly payment on the loan? The principal is $A = \$196{,}000$, the monthly interest rate is $i = 0.054/12 = 0.0045$, and $n = 360$ months. The amortization formula gives

$$d = \frac{\$196{,}000 \times \frac{0.054}{12}}{1 - (1 + \frac{0.054}{12})^{-360}} = \$1100.60$$

Well, that sounds good. But unfortunately there is more to the mortgage than just the amount needed to amortize the loan. Your payment to the bank must also cover real estate taxes and homeowner's insurance on the property. On a $206,000 home, these may add $450 to the monthly payment, which will then total about $1550.

So, no, the median family can't afford the median-priced home, at least not without a bigger down payment or a lower-interest loan. ■

(Norbert Schwerin/The Image Works.)

EXAMPLE 11 Home Equity

My wife's parents sold their house in rural Minnesota to move to the town where we live. They had bought their house in 1980 for $100,000 with a 30-year mortgage at an 8% interest rate. After 22 years, how much **equity** did they have in the house—that is, how much of the principal had been repaid? And how much did they still owe on the house?

SOLUTION What may shock you is that when they sold their house in May 2002—after 269 months of payments, almost exactly three-quarters of the 30 years of the mortgage—they had only $50,000 in equity (hence still owed $50,000 on the house) but had already paid $147,000 in interest. *Three-quarters of their payments had gone to interest.*

We can use the amortization formula to determine just how much equity they had after 269 months of payments, but first we need to determine their monthly payment. We see $A = \$100{,}000$, $n = 360$ months, and $i = 0.08/12$ monthly interest, getting $d = \$733.76$.

Now we use the formula again, this time "in reverse." Knowing $i = 0.08/12$ and $d = \$733.76$, we find out how much of a loan A is paid off by the remaining $n = 360 - 269 = 91$ payments:

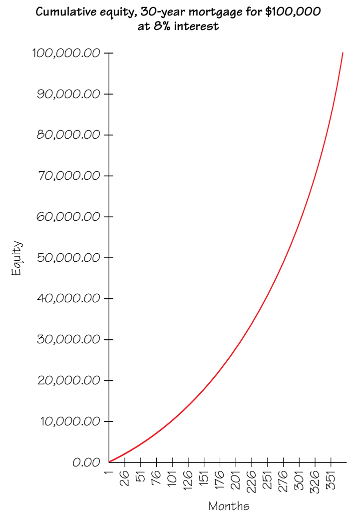

FIGURE 22.1 Equity grows almost exponentially, especially in the later years of a mortgage.

$$A = d\left[\frac{1-(1+i)^{-n}}{i}\right] = \$733.76\left[\frac{1-(1+\frac{0.08}{12})^{-91}}{\frac{0.08}{12}}\right] = \$49{,}940.03$$

This is how much my parents-in-law had yet to pay, so their equity was $100,000 − $49,940.03 = $50,059.97.

Figure 22.1 shows equity versus time for such a loan. The equity grows almost exponentially, but that means that the rate of growth is very slow in the early years of the mortgage. ■

When you buy a home, you have several options for the mortgage: a conventional 30-year mortgage, a conventional 15-year mortgage, or a mortgage for either length of time but with an interest rate that can vary.

You might expect the payment on a 15-year mortgage to be double that of a 30-year mortgage. On the contrary, the payment is only 47% more (for a 5% mortgage) to 26% more (for a 9% mortgage). This range includes the prevailing mortgage rates over the past 20 years. Moreover, over the course of a $100,000 mortgage at 6%, you would pay $164,000 in interest over 30 years but only $72,000 over 15 years. At 9%, the interest totals are $190,000 versus $83,000. (Some financial counselors advise taking a 30-year mortgage and making extra payments when you can afford them, rather than incurring the higher payment obligation of a 15-year loan, which, if you encounter tight personal financial circumstances, you might not be able to afford.) In Spotlight 22.2, we tell what we did in our own circumstances and mention other options.

Since very few mortgages are held for the full term, it is useful to compare the status of mortgages after five years. Table 22.1 shows the equity after five years for a variety of interest rates. For a 30-year mortgage, the equity after five years

SPOTLIGHT 22.2 What We Did with Our House, and What Else You Could Do

We bought our house in 1992. We were offered a choice between an 8.375% fixed-rate 30-year mortgage and an adjustable-rate 30-year mortgage (ARM) at 6.875% whose rate could be raised (or lowered) by up to 2% every year. When we asked, we were also quoted slightly lower rates for corresponding 15-year mortgages.

We were planning to stay in the house much longer than the median of five years, and we were concerned that inflation might force the ARM considerably higher. Also, we did not want the obligation of the higher payments of a 15-year mortgage, in case our circumstances changed (such as through job loss or death). There was no penalty for making extra payments (if we could afford them).

We chose the 8.375% fixed-rate 30-year mortgage (and made some extra payments). Others in other circumstances, or with a different tolerance for risk, would no doubt have decided otherwise. Had we been sure then that interest rates would not go higher in the 1990s, we would have gone for the ARM. But hindsight is always better than foresight. During the early years of this decade, most homeowners with mortgage interest rates such as ours refinanced at much lower prevailing rates, near 5% for a fixed-rate 30-year mortgage.

Currently, about a third of people take ARMs rather than fixed-rate mortgages as one way to respond to soaring real estate prices in some parts of the country. Newer mortgage "products" include interest-only mortgages and shared appreciation mortgages (SAMs). With an interest-only ARM, payments are (just slightly) lower than for a conventional 30-year mortgage, but you accumulate no equity (still, the market value of the house may rise). After five to seven years, you start also paying off the principal—which means that your payments go up then. In some such loans, the interest rate—and your payments—fluctuates as frequently as every month.

In a SAM, interest payments are lower or absent, but the lender receives a portion of any appreciation (rise in value) when the house is sold. In a nationally reported instance in 2003, a single mother received a no-interest SAM loan to finance the $30,000 down payment on a $223,000 house in Pleasanton, California, through the city's affordable housing program. Four years later, she sold the house for $385,000, and the "affordable housing" lenders got 60% of the $162,000 appreciation, or $97,000. She herself realized $65,000 (minus the cost of the sale) but complained bitterly, saying that she would have been better off to have put the loan on her credit card! Critics have termed SAMs an urban form of sharecropping.

TABLE 22.1 Equity on a $100,000 Mortgage After Five Years

Mortgage Term (years)	5%	6%	7%	8%	9%
15	25,700	24,000	22,600	21,200	19,900
30	8,200	6,900	5,900	4,900	4,100

may be less than the cost of selling the home through a realtor. Of course, the resale value of the home may also be higher after five years.

A mortgage with an interest rate that can vary is called an **adjustable-rate mortgage (ARM).** Often such mortgages have a substantially lower interest rate (hence a lower payment) than a fixed-rate mortgage. The ARM's interest rate may go up or down with interest rates in the economy. Usually the rate can be raised or lowered only every year or two, and then by a limited percentage. An ARM may be attractive if you plan to pay off the mortgage after only a few years or because it allows lower payments, it facilitates buying a more expensive home, or you do not plan to keep the home long (hence, you would be selling before the interest rate could rise substantially).

22.4 Annuities

An **annuity** is a specified number of (usually equal) payments at equal intervals of time. We restrict our discussion to *ordinary annuities,* for which payments are made at the end of each interval and the interval is also the compounding period.

An annuity can be interpreted as involving borrowing. For example, winners of lotteries are often offered the choice of receiving either the jackpot amount paid as an annuity over a number of years or else a smaller lump sum to be paid immediately. The cost to the lottery administration is the same. If the winner wants an annuity, the administration buys one from an insurance company for the lump sum. You can think of the insurance company as borrowing the lump sum in exchange for making the payments of the annuity. In effect, the insurance company is amortizing the lump sum over the duration of the annuity.

Winning the Lottery

EXAMPLE 12

On April 16, 2002, three winning tickets shared an almost-record Big Game (since renamed Mega Millions) jackpot of \$331 million. Each ticket's share was one-third of the total, or \$110.333 million, to be paid as an annuity in 26 equal annual installments of \$4.244 million each, the first payment being right away. However, each winner instead chose an instant lump sum of \$58.9 million. What was the interest rate of the annuity?

SOLUTION The insurance company that sold the annuity to the lottery administration regarded \$58.9 million as the present value of the annuity. To consider the annuity as an ordinary annuity, with payments made at the end of each interval, we must subtract the first payment, leaving \$110.333 − \$4.244 = \$106.089 million to be paid in 25 equal annual installments at the end of each year, and \$58.9 − \$4.244 = \$54.656 million as the present value.

In the formula for the present value of an annuity,

$$A = d\left[\frac{1-(1+i)^{-n}}{i}\right]$$

we have $A = \$54.656$, $d = \$106.089/25$, and $n = 25$. To solve for i, the interest rate used by the insurance company, we must use either a calculator with financial mode or a spreadsheet. Either way, we find $i = 5.92\%$. ■

As you save for retirement, it is probably wise to save part of your funds in the form of a tax-deferred annuity (employees of Enron who kept their retirement savings in Enron stock lost big when the company went bankrupt in 2002). If you do not, at retirement you can still sell all of your holdings in other forms and purchase an annuity.

Such annuities differ in a crucial way from the lottery annuity in Example 12. If you retire at 65 and purchase an ordinary annuity, you would be in trouble if you live longer than the term of the annuity (past age 89), because the payments would stop and you would have no further income from the annuity. (About 2% of U.S. children born today can expect to live to age 100.) Similarly, if you die sooner, your estate would still get the payments due after your death, but they wouldn't have helped you meet your living expenses while you were alive.

An approach that avoids these two disadvantages is the *life income annuity:* You receive a fixed amount of income per month for as long as you live. How much you receive per month is based on the life expectancy of people your age, as determined from population data. There are many variations on life annuities, such as payments that increase with anticipated cost-of-living increases, or payments that last until both you and your life partner die (see Spotlight 22.3). But we focus on a simple one-life annuity.

The insurance company that sells you the annuity makes money if you die younger than average and loses money if you die older than average. As in any kind of insurance, over a large number of people, the company can expect gains to balance losses. This is a manifestation of the law of large numbers of Chapter 8. Also, the company's profits vary with the prevailing interest rate during the annuity as compared with the rate built into the annuity.

How much you receive per month depends on your gender. Because women on average live longer than men, the monthly payment to a woman is lower.

EXAMPLE 13 Life Income Annuity

Suppose that you are a 65-year-old male retiring with \$250,000 in a life income annuity. According to the table from one particular insurance company, you would receive \$6.54 per month for every \$1000, so your monthly income would be \$1635. According to the Social Security Administration, your life expectancy at age 65 is about 16.6 years = 199 months. If you lived exactly that long, you would receive a total of $199 \times \$1635 = \$325{,}365$. The rate of interest that your annuity would need to earn to last that long can be calculated from the amortization formula. For example, using =RATE(199, 1635, 0, −250000) in Excel gives a monthly rate of 0.279%, for an effective annual rate of 3.40%.

SPOTLIGHT 22.3 What Actuaries Do

The Truth in Savings Act and the Truth in Lending Act specify that the APY for savings and the APR for loans must be calculated "according to the actuarial method." A loan might not be repaid, and the risk of that happening must be taken into account in setting the interest rate.

Actuaries are financial experts who assess the costs of risks and investigate the probability of various contingencies–for example, death or cancellation–that might occur. Actuaries are crucially involved in setting premiums. Their calculations take into account historical rates–such as the percentage of female 85-year-olds who live to be 86, or the percentage of unmarried male drivers under age 25 who have auto accidents–and project those rates and the accompanying costs into the future.

Other actuaries concentrate on setting up and evaluating pension and fringe benefit plans. For example, the city of Beloit, Wisconsin, hired a consulting actuary to estimate the current and future costs of free lifetime medical benefits to families of police and firefighters.

Another major activity of actuaries is managing return on investment. Contrary to popular belief, insurance companies (particularly life insurance companies) do not earn all of their money from premiums paid. In fact, a substantial portion of their income comes from return on investment of financial *reserves,* funds that they are required to have to meet current and future insurance obligations.

(David Young-Wolff/PhotoEdit.)

Becoming an actuary requires training in mathematics, statistics, economics, and finance, and includes a sequence of professional exams taken over several years.

If you are female and retire now at the same age with the same \$250,000 savings in a life income annuity, you would receive \$6.30 per month for every \$1000, or \$1575 per month. Your life expectancy would be about 19.6 years = 235 months. If you lived exactly that long, you would receive a total of $235 \times \$1575 =$ \$370,125. The rate of interest that your annuity would need to earn to last that long can be calculated from the amortization formula; using =RATE(235, 1575, 0, −250000) in Excel gives a monthly rate of 0.360%, for an effective annual rate of 4.32%. The difference of this figure from that for a man probably reflects the company's use of different values for life expectancy, which vary with region of the country. ■

Notice that a man and a woman who save the same amount receive different monthly incomes at retirement: The woman receives less but for longer–

about 90% as much for 25% longer. Yet their living expenses are likely to be the same. That consideration has resulted in some companies offering "merged gender" rate schedules for annuity payments, so that the individual receives the same monthly payment regardless of gender.

REVIEW VOCABULARY

Add-on loan A loan in which you borrow the principal and pay back principal plus total interest with equal payments.
Adjustable-rate mortgage (ARM) A loan whose interest rate can vary during the course of the loan.
Amortization formula Formula for installment loans that relates the principal A, the interest rate i per compounding period, the payment d at the end of each period, and the number of compounding periods n needed to pay off the loan:

$$A = d\left[\frac{1-(1+i)^{-n}}{i}\right]$$

Amortize To repay in regular installments.
Annual percentage rate (APR) The rate of interest per compounding period times the number of compounding periods per year.
Annuity A specified number of (usually equal) payments at equal intervals of time.
Compound interest formula Formula for the amount in an account that pays compound interest periodically. For an initial principal A and effective rate i per compounding period, the amount after n compounding periods is $A = P(1+i)^n$
Compounding period The fundamental interval for compounding, within which no compounding is done. Also called simply *period*.
Conventional loan A loan in which each payment pays all the current interest and also repays part of the principal.
Discounted loan A loan in which you borrow the principal minus the interest but pay back the entire principal with equal payments.
Effective annual rate (EAR) The effective rate per year.
Effective rate The actual percentage rate, taking into account compounding.
Equity The amount of principal of a loan that has been repaid.
Interest Money earned on a loan.
Nominal rate A stated rate of interest for a specified length of time; a nominal rate does not take into account any compounding.
Principal Initial balance.
Savings formula Formula for the amount in an account to which a regular deposit is made (equal for each period) and interest is credited, both at the end of each period. For a regular deposit of d and an interest rate i per compounding period, the amount A accumulated is

$$A = d\left[\frac{(1+i)^n - 1}{i}\right]$$

Simple interest The method of paying interest on only the initial balance in an account and not on any accrued interest. For a principal P, an interest rate r per year, and t years, the interest I is $I = Prt$.

✔ SKILLS CHECK

1. A city sells bonds in order to

(a) invest money.
(b) raise money.
(c) protect investments.

2. An add-on loan

(a) is a traditional simple-interest loan.
(b) uses compound-interest calculations.

(c) computes interest on the principal for the total loan period.

3. Your car dealer offers to finance a $6000 add-on loan at 3% to be repaid in four years of monthly payments. What is the monthly payment?

(a) $140
(b) $150
(c) $180

4. Repeat Skills Check 3, but the dealer offers a discounted loan instead under the same terms. How much is the loan for if you need to have proceeds (the amount that you get) of $6000?

(a) Exactly $6000
(b) Approximately $6818
(c) Approximately $7200

5. For the same proceeds from a loan (the amount that the borrower gets), the same number of months, and the same rate of interest,

(a) a discounted loan is always cheaper than an add-on loan.
(b) an add-on loan is always cheaper than a discounted loan.
(c) a discounted loan may or may not be cheaper than an add-on loan.

6. Credit-card interest

(a) is computed using compound interest.
(b) is computed using simple interest.
(c) is included in the late fees.

7. If a store credit account charges 1.5% interest each month, what is the effective annual rate?

(a) 18%
(b) More than 18%
(c) Less than 18%

8. If a store credit account charges 1.5% interest each month, what is the annual percentage rate?

(a) 18%
(b) More than 18%
(c) Less than 18%

9. Your credit union offers to finance a $6000 conventional loan at 4% to be repaid in four years of monthly payments. What is the monthly payment?

(a) Approximately $135
(b) Approximately $138
(c) Approximately $145

10. The nominal rate of interest for a loan is

(a) the same as the effective rate.
(b) less than the effective rate.
(c) never greater than the effective rate.

11. If you finance $15,000 for 3 years at 6% compounded monthly, the monthly payments will be

(a) about $456.
(b) about $492.
(c) about $560.

12. If you establish a 30-year mortgage, most of the initial payments

(a) go toward reducing the balance.
(b) go toward paying the interest.
(c) pay insurance costs.

13. After 15 years of payments on a 30-year mortgage, the balance remaining is

(a) about one-third of the original balance.
(b) about one-half of the original balance.
(c) about two-thirds of the original balance.

14. An adjustable-rate mortgage

(a) has variable interest rates but maintains a fixed payment amount.
(b) has variable payment amounts.
(c) is always a better alternative to fixed-rate mortgages.

15. Payments for a 15-year mortgage

(a) are double the payments for a 30-year mortgage of the same amount.
(b) are about 50% to 80% more than payments for a 30-year mortgage of the same amount.
(c) are about 25% to 50% more than payments for a 30-year mortgage of the same amount.

16. Equity in a 30-year conventional mortgage grows

(a) linearly.
(b) logarithmically.
(c) exponentially.

17. A convenient rule of thumb is that for a 30-year mortgage at 6%, the monthly payment is about 0.6% of the loan. So, on a $100,000 mortgage, the monthly payment is about $600. How much of the first payment goes toward interest?

(a) All of it
(b) About $100
(c) About $500

18. Which of the following arrangements could be an ordinary annuity?

(a) Monthly payments, annual compounding
(b) Annual payments, monthly compounding
(c) Annual payments, annual compounding

19. If you just won a lottery jackpot paid in 25 equal annual installments of $1 million each, what is the present value for the jackpot?

(a) $25 million
(b) More than $25 million
(c) Less than $25 million

20. A life income annuity is designed to pay a fixed amount each period until

(a) the annuity runs out of money.
(b) you die.
(c) you reach your life expectancy.

EXERCISES

■ **Challenge** ◆ **Discussion**

Simple Interest

1. On August 13, 2001, you could buy $10,000 U.S. Treasury bonds that pay simple interest of

- 5.51% per year in February each year through maturity in February 2031, at a cost of $9,802 each; or
- 11.25% per year in February each year through 2015, at a cost of $15,529 each.

Each bond also returns $10,000 at maturity.

(a) What is the annual yield of each investment?
(b) Compare these two investment opportunities, which differ vastly in interest rate but also in cost.

2. In late September 2001, you could buy a U.S. Treasury bond offering 5.75% interest and maturing one year later. The face value of the bond was $10,000; this is the amount of the principal that the Treasury pays back to the owner at maturity. But resellers of this particular bond wanted $10,318. On this bond, what was the *current yield*—the annual percentage return to the purchaser at the current price?

3. You need to buy a car and need to finance $5000 of the cost. The dealer offers you a 5.9% add-on loan to be repaid in monthly installments over four years. How much is your monthly payment?

4. You have to make some home improvements—well, to be honest, they're really maintenance that you can't defer any longer!—and need to borrow $3000 to pay for them. You can get an 8.5% add-on loan from a savings and loan association to be repaid in installments over two years. How much is your monthly payment?

5. You are in the same situation as in Exercise 4, except that you find that you can get a 9% discounted loan from a loan company to be repaid in monthly installments over four years. What is the monthly payment on this loan?

6. You are in the same situation as in Exercise 4, except that you find that you can get an 8.5% discounted loan from a loan company to be repaid in monthly installments over five years. What is the monthly payment on this loan?

7. Suppose that you need $1000 (no less) and have available to you either an add-on loan or a discounted loan, both at the same interest rate and for the same period. Which will have the lower monthly payment?

8. Show algebraically that your claim in Exercise 7 is true in general for a loan of any amount.

Compound Interest

9. A credit-card bill of mine showed \$500 due, with a minimum payment of \$10 and daily interest rate of $r = 0.04932\%$. If I make no more charges on the card and pay \$10 a month as soon as I get each bill, how long will it take to pay off the total? (*Hint:* The amortization formula can be changed algebraically into the form

$$(1 + i)^n = \frac{d}{d - iP}$$

Note that making the first payment immediately, will reduce the principal P to be amortized to \$500 − \$10 = \$490. If I delay payment, I incur additional daily interest on the amount due. Evaluate the right-hand side. Then, using either a spreadsheet or the power key $\boxed{y^x}$ on your calculator, raise the value of $(1 + i)$ to higher and higher powers n until you find the smallest value for n that makes the left-hand side larger than the right-hand side.)

10. (Requires spreadsheet) Regarding the credit-card bill in Exercise 9: By paying \$10 each month, approximately how much interest would I have paid by the time I pay off the original \$500? (*Hint:* You won't be off by much if you estimate the last payment to be \$10.)

11. (Requires spreadsheet) According to the regulations for the credit card discussed in Exercises 9 and 10, the minimum payment is supposed to be the greater of \$10 or 2% of the balance (rounded down to the next higher dollar amount). Suppose that you have such a card and the balance is \$1500. Neglect the rounding down and assume that you pay exactly 2% of the current balance each month until the balance reaches \$500. Notice that if you make a payment of 2% and the bank charges daily interest of 0.04932% in each 30-day month, you in effect reduce the balance to $(1 - 0.02)(1.0004932)^{30} = 99.46043\%$ of what it was the previous month. How many 30-day months will it take to reduce the balance of \$1500 to \$500? (Again, assume that you make payments right away.)

12. By making the minimum payment each month in the situation that exists in Exercise 11, approximately how much interest will you have paid by the time you pay off the original \$1500?

13. Adding your answers from Exercises 9 and 11, determine how long it will take to pay off a balance of \$1500 by making minimum payments.

14. (Requires spreadsheet) Assume the same situation as in Exercise 11, but instead of paying exactly 2% of the balance due, you make a minimum payment of the greater of \$10 or 2% of the balance (rounded to the next higher dollar amount).

Conventional Loans

15. In September 2004 you could buy a new Kia Amanti car for \$24,995 with a loan from the manufacturer at 0% (!) interest over 60 months. Assume that you had cash or a trade-in worth \$2000. What was the monthly payment?

16. You can't afford the new car in Exercise 15 but need a better-than-junk car, and you need to borrow the entire \$7000 of its cost. You can get a 48-month conventional loan–not an add-on loan or a discounted loan–from your credit union at 8.5% compounded monthly. What is your monthly payment?

17. A television ad that ran in February 2002 advertised a new 2002 Pontiac for 16% down with a five-year loan at 5.9%, with a payment of \$19.09 per month for each \$1000 financed. Check whether the monthly payment is correct (I may have miscopied it from the fine print that showed for three seconds at the end of the ad!).

18. For the same car mentioned in Exercise 17, a car dealer offered a 7.9% APR conventional loan over 48 months but a 6.8% APR conventional loan over 60 months.

(a) What is the payment per \$1000 under each loan?
(b) With the 60-month loan, it takes longer for the dealer to be paid in full. So what is the

advantage to the dealer in offering a lower percentage loan for 60 months, instead of a higher percentage loan for 48 months? (Thanks to Terence Blows of Northern Arizona University.)

19. Suppose that you have good credit and can get a 30-year mortgage for $100,000 at 6.5%. What is your monthly payment?

20. Assume the same situation as in Exercise 19, except that your credit is not as good and the rate that you are offered is 7.125%.

21. Assume the same situation as in Exercise 19, but you inquire about a 15-year loan instead. You are offered 6.125%. What is your monthly payment?

22. Assume the same situation as in Exercise 21, but your credit is not as good, and you are offered 6.75%. What is your monthly payment?

23. For the mortgage in Exercise 19, how much equity would you have after five years?

24. For the mortgage in Exercise 20, how much equity would you have after five years?

25. For the mortgage in Exercise 21, how much equity would you have after five years?

26. For the mortgage in Exercise 22, how much equity would you have after five years?

27. Despite a filter, lots of spam gets into my email. For a while I was getting mortgage offers, such as "$160,000 for less than $735 per month" (for a 30-year mortgage). What would be the corresponding interest rate?

28. At about the same time as the offer in Exercise 27, I was getting spam offers for 3.6% mortgages. In terms of the monthly payment, would this be a better deal than the mortgage in Exercise 35? The 3.6% was about 2% below the going rate at local banks. What would you guess might explain the difference?

29. Suppose that you and two friends decide to live off-campus in your senior year. One of them (who has wealthy parents) suggests that instead of renting an apartment, you could buy a house together, live in it for your senior year, then rent it out or else sell it. Assuming that (with the help of her parents and their good credit rating) you could get a mortgage for $180,000 to buy a house near the campus, what would be the monthly mortgage payment on a 30-year mortgage at 6.75%?

30. For the add-on loan in Exercise 3, what is the APR? What is the EAR?

31. For the add-on loan in Exercise 4, what is the APR? What is the EAR?

32. For the discounted loan in Exercise 5, what is the APR? What is the EAR?

33. For the discounted loan in Exercise 6, what is the APR? What is the EAR?

34. For the credit card in Exercises 9 and 10, what is the APR? What is the EAR? What annual rate must the credit-card company tell its customers applies?

For Exercises 35 and 36, refer to the following.

Payday lenders provide short-term loans, usually for two weeks, until the borrower's next payday, in one of two forms:

- The borrower writes a postdated check to a lender and the lender advances the cash, minus a finance charge; this is in effect a discounted loan.
- The borrower signs an authorization for the lender to debit the borrower's bank account on payday for the amount of the loan plus the finance charge. This is in effect an add-on loan with a single installment payment.

The industry grew in the 1990s out of check-cashing services to approximately 12,000 firms in 31 states and the District of Columbia, with a revenue of over $2 billion by 2000. Comptroller of the Currency John D. Hawke, Jr., noted that "California alone has more payday loan offices—nearly 2,000—than it does McDonalds and Burger Kings."

The average loan is $300, for two weeks, with an average fee of $54.

35. For the discounted loan format, and average loan size and fee, what is the APR?

36. For the add-on loan form, and average loan size and fee, what is the APR?

For Exercises 37 and 38, refer to the following.

Many income tax preparation services, including the large national chains, offer refund anticipation loans (RALs), or "rapid refunds." These are similar to payday loans in providing an advance on anticipated income–in this case, a tax refund. The loan is repaid when the IRS pays the refund, usually about 7 to 17 days after the loan is made. The cost of the loan is deducted from the proceeds to the client, so this is a discounted loan. The RAL business takes in about $2 billion each year (including tax preparation and check-cashing fees) to arrange payment of the earned-income tax credit to working parents, about 7% of the total of this aid to poor families. A RAL for an anticipated refund usually is issued for a flat fee, often $88, and the average loan is $1500.

37. Suppose that the RAL speeds the refund by 7 days. What is the APR for the average RAL?

38. Suppose that the RAL speeds the refund by 17 days. What is the APR for the average RAL?

39. When interest rates drop, it may become attractive to refinance your home. Refinancing means that you acquire a new mortgage to borrow the current principal due on your home and use the proceeds to pay off your old mortgage. You then begin a new 15- or 30-year mortgage at the new, lower interest rate. A second factor that reduces your monthly payment is that the equity you accumulated under the old mortgage reduces the amount that you have to borrow under the new mortgage. Suppose that you have an existing 30-year $100,000 mortgage at 8.375%, on which you have been paying for five years, and you are considering refinancing at 7.0%.

(a) What is your payment under the old mortgage?
(b) How much equity do you have in the home?
(c) If you use all your equity to reduce the amount of the new mortgage, how much will your monthly payment be?
(d) How long is the payback period for the $2000 loan charge–that is, how many months will it take before you have saved $2000 in monthly payments?

40. (Requires spreadsheet) Even if interest rates go up, you may find it beneficial to refinance if your financial circumstances suddenly worsen. Consider an actual individual (as recounted in the *Wall Street Journal* of October 30, 2001), a 39-year-old plumber whose income was cut in half by worsening economic conditions. She had missed eight monthly mortgage payments totaling $14,000, still owed $160,000 on the mortgage, and was in danger of foreclosure and losing her home. Data on her loan were not given in the *Journal,* but data that fit are a 15-year mortgage for $180,000 at 8.375%.

(a) What was her monthly payment?
(b) How long had she been paying on the mortgage?
(c) Under a "loan modification" program, the bank added the $14,000 in missed payments to her existing debt of $160,000. Suppose that the bank offered her a new 30-year loan for the total of $174,000 at 8.75% (despite interest rates in late 2001 being generally lower than when she got her first loan, her delinquency on that loan made her a worse risk and hence she incurred a higher interest rate). How much was her new monthly payment?

■ **41.** Fisher's effect (Chapter 21, p. 828) gives the real rate for borrowing, sometimes called the *real cost of capital.* Suppose that you buy a house with a $100,000 mortgage at an interest rate of 6.75% and that inflation remains at 3% for the duration of the mortgage. You make monthly payments in constantly deflating dollars. Simplify the situation by supposing that you sell the house one month after you buy it (the neighbors really drove you crazy, right from the start).

(a) What would the house sell for if it kept up with inflation of 3%/12 = 0.25% per month?
(b) Calculate your mortgage payment at the end of the month. How much is interest and how much is principal?
(c) After making the mortgage payment, paying off the balance of the $100,000 cost of the house, and selling the house for the amount in part (a), how much did it cost you to own the house for the month?
(d) The cost in part (c) is in deflated dollars. Using a formula from Chapter 21, convert this to constant (month-before) dollars and divide by $100,000 to express the result as a percentage rate. This is the real rate of interest.

42. One of the advantages of buying a home with a fixed-rate mortgage is that your payment stays the same but your earnings and the value of your home are likely to go up with inflation. You are paying back the loan with dollars of lesser value.

Consider the following scenario. Suppose that you buy a "starter" two-bedroom home for $105,000 under a special program for first-time home buyers that requires a down payment of only $5000. You have a 30-year fixed-rate mortgage for $100,000 at 7%, on which the monthly payment is $665.30. You also have a $2000 one-time expense in closing costs and annual costs of $200 for insurance and $2000 for property taxes.

You live in the home for five years and spend $10,000 on maintenance, upkeep, and improvements. You then sell the home for $125,000, pay a realtor $9000 to sell it, and pay closing costs of $500 (for title insurance and other costs). Finally, it costs $3000 to move.

(a) Make out a balance sheet of revenue and expenses. How did you make out on owning the home?
(b) Remember, you also got to live in the home without paying rent! Translate the cost of owning the home into an equivalent monthly rent.

Annuities

43. The largest amount won by an individual in the Florida Lotto lottery was $81.6 million on March 29, 2000 (other jackpots have been higher but were shared). The advertised jackpot amount is paid in an annuity of 30 annual payments, including one immediate payment.

Instead of the 30-year payment plan, the winner may choose to receive a smaller lump sum right away in cash. If not, that sum (minus the initial payment) is invested by the State Board of Administration in U.S. government securities on behalf of the player. The cash invested, plus all interest earned, goes to the winner.

Suppose that in April 2000 the state of Florida could buy U.S. securities paying an interest rate of 7%. What was the present value of the jackpot annuity—that is, how much could the winner have received in cash instead? (We say that $81.6 million is the *future value* of the annuity.)

44. Suppose that you retire at age 65 and in addition to Social Security need $2000 per month in income. Based on an expected lifetime of 16.6 more years (for men) or 19.6 more years (for women), how much would you have to invest in a life income annuity earning 4% to pay you that much per year?

45. (Spreadsheet helpful) We return to your forward-thinking roommate of Exercises 41 and 42 in Chapter 21 (p. 841), who has already planned her retirement. She now wants to retire at 55 in 2040 with a steady income of $50,000 a year in 2005 dollars. Her plan is to amass a fortune and live off just the interest. But why not live it up and spend every last dollar? You suggest to her that instead she plan to buy in 2040 a special kind of 45-year annuity (in case she lives to 100!) whose payments increase 4% each year. (Recall, she assumes 7.2% APR on her investment and 4% inflation.) What will such an annuity cost? How much will she have to invest quarterly between 2005 and 2040? (*Hint:* What is the present value in 2040 dollars of the annuity payments?) (Under this plan she celebrates her 100th birthday by going broke!)

46. (Spreadsheet helpful) Your other roommate also started a retirement fund in 2005, but she took a few other factors into account, too (she plans to be an actuary). She aims to start retirement with $50,000 a year in 2005 dollars in 2052 when she is 67, the age at which she can receive government health insurance (Medicare) and full Social Security payments. She consulted the Social Security Administration, which estimates that a medium-earning person retiring then can expect to receive $1628 per month in 2001 dollars. She estimates (more conservatively) a steady 6% APR on her investment and a steady 3% annual inflation. She plans to live to 100 (why not try?) and to buy a 32-year annuity in 2052 (also paying 6%). The annuity would supplement Social Security payments to give a total the first year of $50,000 in 2005 dollars, and the same *current* dollar amount each year of the 32 anticipated years of retirement. What must the size of her nest egg be, and what should be her monthly investment over the 47 years, to achieve her goal?

47. Plan as we may, the future is not certain. The roommate in actuarial studies knows that various plans are afoot to "bail out" Social Security. One such plan would raise the age for full Social Security benefits to 68 and reduce those benefits by 32.5% for those retiring in 2052. As in Exercise 46, calculate how much she should accumulate (over 48 years) to meet her financial goal for the remaining 31 years until she reaches 100.

APPLET EXERCISES

To do these exercises, go to www.whfreeman.com/fapp7e.

There are two ways to buy a car: save up and pay cash or borrow the money from a bank. In the *Buying a Car: Cash vs. Loan* applet, you can explore just how much more expensive it is to borrow the money.

WRITING PROJECTS

1. In recent years, incentives from auto manufacturers to potential customers have taken the form of offering either a reduced interest rate on the loan for the car or else a rebate (reduction in price) on the cost itself. In fall 2004 you could buy a 2005 Toyota Camry for $13,570, with one of the following options for payment:

- $750 rebate
- 1.9% APR over 24 months
- 1.9% APR over 36 months
- 2.9% APR over 48 months
- 3.9% APR over 60 months

Suppose that you could afford a $2000 down payment and could get a loan from a credit union at 6.0% over 60 months if you opt for the $750 rebate.

(a) What is your monthly payment under each option?
(b) Suppose that the rate of inflation over the course of the loan is a steady 3% per year. How do the various options compare in terms of present value of the loan?
(c) Locate current advertised incentives for a car that you would like to buy and compare them in an essay of two to three pages.

2. A substantial proportion of new cars today are not sold but leased. Contact a local car dealer about a car that you are interested in and find out the details on leasing. Compare the cost of the lease and associated expenses with the cost of purchasing and owning the car. Include estimated maintenance, repair, and insurance costs for each option. Which seems like a better deal, and why? Write two to three pages describing and comparing the two options.

3. Banks typically offer mortgages with various combinations of interest rates and points. Loans at a higher interest rate tend to come with more points, and vice versa. Like interest, points are deductible on income taxes, but in the year in which the loan is made. In recent years, home buyers have (when offered the option) generally favored paying fewer points. Suppose that you have a choice between a mortgage at 6% with 2 points (2%) and a mortgage at 8% and no points. Which would you choose, and why? Does it make a difference how long you are planning to own the home? Or how expensive the home is? Write a page justifying your decision.

4. Explore actual costs of homes in your area, mortgages with local banks (including closing costs), and property taxes and insurance. Come up with data such as those in Exercise 40 and make out a corresponding balance sheet for five-year ownership.

SUGGESTED READINGS

KASTING, MARTHA. *Concepts of Math for Business: The Mathematics of Finance* (UMAP Modules in Undergraduate Mathematics and Its Applications: Module 370–372), COMAP, Inc., Arlington, Mass., 1980.

MILLER, CHARLES D., VERN E. HEEREN, and JOHN HORNSBY. Consumer mathematics. In *Mathematical Ideas,* 9th ed., Addison Wesley Longman, Boston, 2001, pp. 786–845.

VEST, FLOYD, and REYNOLDS GRIFFITH. The mathematics of bond pricing and interest rate risk, *Consortium* (COMAP) no. 59 (Fall 1996): HiMAP Pullout Section 1–6.

YAREMA, CONNIE H., and JOHN H. SAMPSON. Just say "Charge it!" *Mathematics Teacher* 94 (7) (October 2001), 558–564. Shows how to apply the savings formula and the amortization formula and graph the results on the TI-83 calculator. Notes that the 78% of undergraduates in the United States who have credit cards carry an average debt of more than $2700, with 10% owing more than $7000.

SUGGESTED WEB SITES

http://www.lendingtree.com/stmrc/calculators1.asp Java applet calculators (for any platform) to calculate payments and amortization schedules for conventional loans, adjustable-rate mortgages, auto loan vs. home-equity loan, and credit-card payoff. (*Note:* Lending Tree, Inc., is a loan broker; mention here of calculators at its Web site does not imply endorsement of its other services by this book's authors, editors, or publisher.)

http://www.udayton.edu/sba/wf/wksht/wksht5.htm Excel spreadsheet templates for comparing consumer loans, tax-deferred retirement plans, mortgage loans, present value, lease vs. buy, and more.

http://www.edmunds.com/apps/calc/CalculatorController?pmtcalAction=apr_cash_calc Commercial site offering a calculator to compare rebate vs. interest-rate offers for car purchase. (*Note:* Edmunds is a loan broker; mention here of calculators at its Web site does not imply endorsement of its other services by this book's authors, editors, or publisher.)