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The Sun's Position at Midday and the Analemma

The Sun's position at midday at any site will not always be on the observer's meridian. One reason for this is that the site may be offset from the center of the time zone (i.e., the site is not on the central meridian of the time zone); this produces an offset of the Sun from the observer's meridian at noon that remains constant throughout the year. If time zone boundaries were ideal, then this offset could be as large as 1/2 hour; but in fact time zone boundaries sometimes deviate from ideal for political reasons, resulting in offsets of up to about an hour for some locations.

In addition to this constant offset, there is also a varying deviation of the Sun from the meridian that is caused by the combination of two effects. The first of these is the Earth's elliptical orbit, which results in a variable speed of the Earth around the Sun, as described by Kepler's second law. This law is described in Section 4-4 of Kaufman and Freedman, *Universe*, Fifth Ed. The changing orbital speed of the Earth around the Sun changes the apparent speed of the Sun across our sky, day by day. The second effect is the tilt of the Earth's spin axis to its orbital plane, also called the ecliptic plane. This also causes a variation in the speed of the apparent motion of the Sun.

The varying deviation of the Sun east or west described in the previous paragraph, when combined with the annual motion of the Sun north and south due to the seasons, results in a figure-eight pattern for the Sun's midday position in our sky over the course of a year. This figure-eight pattern is called the **analemma**, and is often found on globes representing the Earth. This truly amazing multiple-exposure photograph required a year to take, at a rate of one exposure approximately every 8 days!

The shape of the analemma became important when sundials were used to tell the time by tracing the movement of the shadow of a vertical post upon the ground. The analemma provided corrections, amounting at times to 16 minutes, to times determined by this shadow at different times of the year.

The present exercise demonstrates the analemma by tracing the apparent motion of the Sun in the sky at midday for a whole year at a chosen site.

PART 1. Finding the Shape and Size of the Analemma

A. Screen Set-up

1. After starting the program, check the icon/location box to see that the field of view is 100° (if this box is not visible on the screen, you can activate it by clicking Show Floating Palettes in the Window menu). If the field of view is not 100°, then click on the house button to reset the field of view to 100°.

2. Set the location to Calgary, Alberta, Canada, near 50° latitude (Settings/Viewing Location/North America/Canada/Calgary/Set Location). At this latitude the Sun is always visible on the screen at noon; that is, it is above the horizon in midwinter and not out of view above the top of the screen in midsummer.

3. If your viewing direction is not already toward the south (S), then click the S button on the button bar near the top of the screen. If this button bar is not visible, activate it by clicking

Toolbar in the Window menu. After the screen adjusts direction, you should see the symbol S on the horizon near the bottom of your screen.

4. Check that the Daylight Saving Time option is turned off. To do this, check the small sun icon to the left of the time in the date/time box (if this box is not visible on the screen, activate it by clicking Time in the Window menu). The sun icon is yellow with 8 rays if DST is activated and background color with 4 rays if DST is not activated. If the icon is yellow, click on it to deactivate DST.

5. Stop the time by clicking the time-stop button in the date/time box; then
   a. set the date to the winter solstice, December 21, 2000 AD (12/21/2000 AD);
   b. set the time to midday, 12:00:00 PM;
   c. set the time interval in the date/time box to 7 days. (Make sure you use "days," which actually means solar days and not "sidereal days," since you want to trace the Sun's apparent motion at midday every 7 solar days.)

6. Switch off daylight by clicking on Daylight in the Display menu. This will enable you to see the stars moving across the daytime sky, week-by-week.

7. Check that the cursor is the Grabber Tool (hand icon). If it is not, then select the Grabber Tool by clicking on the hand icon in the icon/location box. If this box is not visible on the screen, you can activate it by clicking Show Floating Palettes in the Window menu. Alternatively, you can activate the Grabber Tool by pressing the F4 key.

B. Tracing the Analemma

After the above set-up, the Sun should be at a low angle above the southern horizon. You will notice that the Sun is *not* due south at this time. This is because Calgary is offset from the center of its time zone, the Mountain Standard Time zone, by about 36 minutes. You can adjust for this offset, if you desire, by changing the local time to 12:36:00 PM.

The shadows of the trees are cast across the ground before you. You can think of these trees as representing a sundial for measuring time at this location. The position of the Sun in the sky will move these shadows around, as we shall see.

You need to move your viewpoint so that the Sun is just above the bottom of the screen at this time so that you can follow the path of the Sun through the full year. To do this, use the Grabber Tool to move the sky downward (click and hold the left mouse button while sliding the mouse toward you).

In order to trace the motion of the Sun week by week, you need to tape a piece of transparent plastic or tracing paper to your screen. Mark the corners of your screen on the transparent sheet with a felt pen or marker. Be very careful not to damage the screen when you do this. Mark the Sun's position, then single-step the time forward by 7 days and mark the Sun's position again. Continue this process for a whole year of time, tracing out the Sun's position, or the analemma. Leave the plastic or tracing paper where it is for the next step.

Switch on the equatorial coordinate grid (click Equatorial Grid in the Guides menu or press CTRL+2), and mark the Right Ascension grid lines on either side of the analemma and all of the declination grid lines in the vertical direction. These are 1-hour right ascension and 10° declination intervals. Now remove your plastic film or tracing paper from the screen so that you can make measurements on it.

Questions

1. The reason for the north–south motion of the Sun is the tilt of the Earth's axis to the ecliptic plane. Using the declination lines on your plot, estimate this tilt by measuring the maximum N–S excursion of the Sun in degrees of declination over a full year. This will be twice the tilt-angle of the Earth's axis. It is this change in the angle of the Sun in the sky that produces seasonal changes upon the Earth.

2. Estimate the maximum error in a sundial from the width of your plot, assuming that the Sun would be "on time" if it were on the center line that runs lengthwise through the analemma. This makes the sundial error equal to one half of the width of your analemma. For reference, one interval of Right Ascension on the screen is equal to 1 hour of time, or 60 minutes, since one hour of Right Ascension is the angle through which the Earth rotates in 1 hour.

3. At what times of the year will the sundial be most accurate?

4. At what times of the year will the sundial be most inaccurate?

PART 2. The Analemma from Other Latitudes

The shape of the analemma will be the same from any position on Earth. You can test this hypothesis by moving to another location, say Sao Paulo, Brazil (Settings/Viewing Location/South America/Brazil/Sao Paulo/Set Location). Reset the date to 12/21/2000 AD, and the time to 12:00:00 PM. Check that daylight savings time is off. Remember that the Sun will now be in the northern sky from the southern hemisphere so click on the N position on the top toolbar. In order to see the Sun, you will have to use the Grabber Tool to move the sky down so that the Sun is just visible at the top of the screen at this time. Now set time running at 7-day intervals to see the analemma for Sao Paolo. As you see, it is the same shape as it was from a northern latitude site.

PART 3. Sunrise Time at the Winter Solstice

A question that is sometimes asked is, "If the Sun is at its furthest South at the time of solstice, on the shortest day of the year, December 21, why is this *not* the date of latest sunrise?" In fact, the date of latest sunrise is close to the last day of the year, several days later!

In the steps below, you can answer this question in terms of the analemma and the position of the Sun as it moves through the time of the solstice.

A. Screen Set-up

1. Set up again for Calgary, Canada (Settings/Viewing Location/North America/Canada/Calgary/ Set Location).

2. Reset the date to 12/21/2000 AD.

3. Set the time to 8:42:00 AM.

4. Set the time interval to 1 solar day (the time-step labeled "days").

5. Check that Daylight Saving Time is off.

6. Check that daylight is off.

7. Move to the E position using the top toolbar. You should see the Sun on the horizon near the SE position.

8. In order to demonstrate this sunrise effect, it is best to magnify the view of the sky in the region of the Sun. To do this, use the Magnification Tool (activate the Magnification Tool by clicking on the magnifying glass icon in the icon/location box, or press F8). Place the Magnification Tool on the Sun and click several times until the Field of View is 4°, as shown by the number in the lowest box in the icon/location box. Be sure the Magnification Tool is on the Sun each time. If you lose the Sun from the screen at any magnification, then reverse the magnification by holding down the control key (CTRL) while you click the mouse, until you find the Sun again; then re-magnify by clicking on the Sun. When you are finished, return to the Grabber Tool (click on the hand icon in the icon/location box or press F4).

9. You should see the Sun near the center of the screen, just touching the horizon. If necessary, use the Grabber Tool to center the Sun on the screen. If the lower edge of the Sun is well below the horizon or above the horizon, temporarily set the time interval to 10 seconds and step forward or backward to place the Sun at a position just touching the horizon; then reset the time interval to 1 day.

B. Date of Latest Sunrise

1. You can see that on the winter solstice the Sun is just rising at Calgary at the time shown on the screen. Now, advance the time by 1-day steps. You will see that, for several days after December 21, the analemma motion of the Sun places it further below the horizon at 8:42:00 AM than it was the previous day. This means that sunrise will be later on these days than on the shortest day.

2. Experiment to find the date of latest sunrise, and record this date.

3. Set the date to the latest sunrise date, then change the time interval to 10 seconds and step time until the Sun is just clear of the horizon. Record this time.

Questions

1. On what date does the latest sunrise occur?

2. About how many days after the winter solstice does the latest sunrise occur?

3. By how many minutes in the day is this sunrise later than sunrise at the solstice?

Answers

Tracing the Analemma

1. Tilt of Earth's axis = 23 1/2°

2. About 14 minutes.

3. At the solstices (mid-summer and mid-winter), and around May 20 and October 26.

4. About March 11 and October 10.

Date of Latest Sunrise

1. About December 29.

2. About 8 days after the solstice.

3. 2 minutes, 20 seconds.