More

Delay of Light in a Gravitational Field

Einstein was led to develop general relativity because Newton's theory of gravitation could not be included within the framework of special relativity. General relativity solved that problem, as well as a number of conceptual difficulties with Newton's theory. Among these difficulties are classical theory's exclusion of zero-rest-mass particles and the fact that the classical gravitational force was an *action at a distance* (i.e., there was no contact between the masses involved) transmitted instantaneously (i.e., with infinite speed), in conflict with the special relativistic limitation of the speed of light as the maximum speed for transmission of signals. General relativity was able to include gravity by using the principle of equivalence to replace the gravitational field, including inhomogeneous fields, at *every* point in space with an appropriately accelerated local reference frame. In each of these local reference frames special relativity is valid and Einstein's first postulate, the principle of relativity, applies. The result of all of this, which involves mathematics well beyond the scope of our discussions, is to produce the modified spacetime interval Δs (Equation 2-43) that we used in two space dimensions earlier:

$$(ds)^2 = \left(1 - \frac{2GM}{c^2 r}\right)(c\,dt)^2 - \left[\frac{(dr)^2}{(1 - 2GM/c^2 r)} - r^2\,d\theta^2 - r^2 \sin^2\theta\,(d\phi)^2\right] \qquad \textbf{2-52}$$

This expression links gravity, embodied by the $(1 - 2GM/c^2r)$ term, to the coordinates or the geometry of spacetime. Notice that if $M = 0$, then $(ds)^2$ has our old familiar form, but if $M \neq 0$, then $(ds)^2$ changes too. Qualitatively, associating ds with the slope, $(ds)^2$ becomes the equivalent of the curvature of spacetime. Thus, we arrive at the idea that changing the mass in a particular region changes the curvature of spacetime in that region.

This exceedingly brief, qualitative discussion hardly does justice to the topic, but it does allows us to plausibly draw a two-dimensional analogue of the effect of mass on the curvature of spacetime. Illustrating what is often called the "rubber sheet analogy," Figure 2-22*a* shows the path of a light beam A and the trajectory of a particle with (small) rest mass B moving through a region of two-dimensional spacetime where there are no large masses. The light moves in a straight worldline at speed c with $ds = 0$; the particle's worldline is also straight (with $ds \neq 0$) in the absence of external force. The addition of the large mass M in Figure 2-22*b* distorts or *warps* spacetime. The light and the particle cannot, of course, leave spacetime and now move along the paths A' and B'. The particle's trajectory shows the "gravitational attraction" of the mass M, but not as some mysterious action-at-a-distance force. Instead, the particle is simply moving in the *straightest possible line* (the so-called *geodetic*) in the warped space. The light beam does exactly the same thing!

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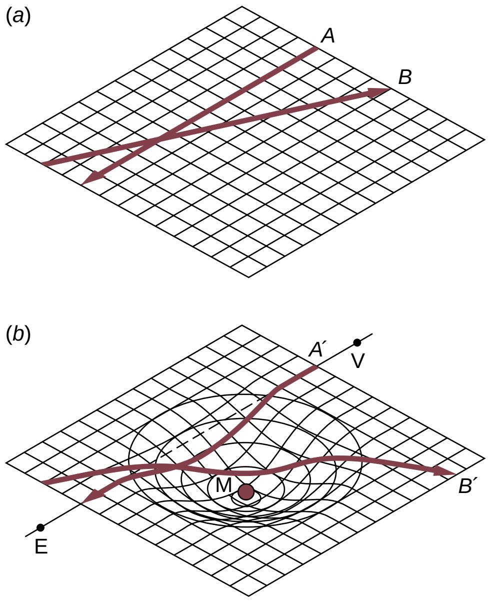

Fig. 2-22 The "rubber sheet analogy" shows a two-dimensional spacetime without a large mass (*a*) and with a large mass *M* (*b*). *A* and *B* are the worldlines of light and a particle with mass, respectively. E and V show Earth and Venus as they were during one of Shapiro's measurements of the time delay of a radar signal.

Thus, the worldline of light is now curved, and that makes the light appear to slow down to observers relatively far from *M*. For example, suppose that *M* in Figure 2-22*b* is the sun and E and V are, respectively, Earth and Venus. Ignoring the masses of the two planets compared with *M*, when they are on opposite sides of the sun (called being in *superior conjunction* by astronomers), the path of light moving between them would be slightly longer than the "direct path," i.e., the path without the presence of *M* warping local spacetime. The distance between the planets in this situation can be accurately computed from their orbits as determined from Newton's law of gravity; hence, the time needed for light to travel between the two can be found. A round trip from Earth to Venus to Earth would require about 20 min. Since the worldline of the light will be slightly longer with the sun present, the light

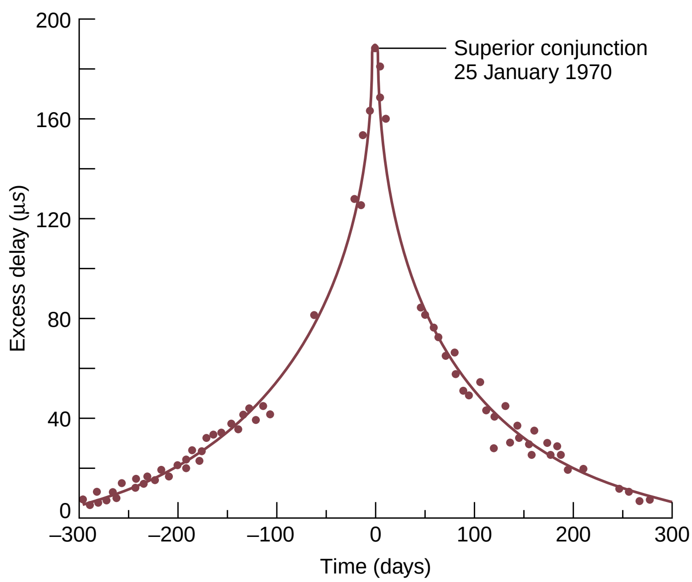

Fig. 2-23 Time delay in the round trip of radar signals reflecting from Venus. The maximum delay, when the sun's edge touches the line between Earth and Venus, is 200 μs. The solid line is the prediction of general relativity. [*After I. I. Shapiro et al., 1971,* Physical Review Letters, **26,** *1132 (1971).*]

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will appear to be delayed, or slowed down, if general relativity is correct. It would therefore take the light a bit longer than 20 min for the round trip. The delay should depend on how close to the sun the light passes, i.e., on how much spacetime is curved along its worldline.

In 1971, I. I. Shapiro and his co-workers[23] reported on a highly successful series of experiments in which radar signals were reflected from Mercury, Venus, and Mars as each moved on the far side of the sun from the Earth. Shapiro's data for the measured delay as Venus approached and receded from superior conjunction is shown in Figure 2-23. The solid curve in the figure is the prediction of general relativity. The agreement of the data with the theory is apparent, but also remarkable when one recognizes that the uncertainty of the data, $\pm 20\ \mu s$, requires that the relative positions of the two planets be known to within a few kilometers.